

Table VI. Comparisons of ϕ_K Obtained in this Study and the Data of Millero et al. at $I_m = 1.0$, 0°C , and 1 Atm

Salt	$-10^4\phi_K$ (ours)	$-10^4\phi_K^a$ (Millero et al.)
NaCl	78 ± 28	60.4
MgCl ₂	143 ± 22	121.2
Na ₂ SO ₄	219 ± 29	171.4
MgSO ₄	213 ± 24	156.4

^a Millero et al. (16), eq 10.

et al. (16) (Table VI). The result of MgSO₄ is higher than the data of Millero et al.

Currently, the atmospheric densities and heat capacities of the major sea salts are being measured over a wide temperature range in our laboratory. Also the high pressure sound speed in the aqueous solutions of the major sea salts is being measured; these new results will be combined with the atmospheric densities and heat capacities to give precise equations of state for the major sea salts. The apparent molal volumes and compressibilities calculated from the sound-derived equations of state will be compared with the results of this study.

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Preliminary Data on the Pressure Effect on the Viscosity of Sodium Chloride–Water Solutions in the Range $10\text{--}40^\circ\text{C}$

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The paper describes new, and preliminary, measurements of the viscosity of NaCl solutions in H₂O over a range of temperatures $10\text{--}40^\circ\text{C}$, a range of pressures $0\text{--}30.0$ MPa, and at three concentrations, together with check measurements on pure water. The precision is 0.2% and the accuracy is 0.3%. The measurements are represented by a purely empirical, analytic correlation which reproduces the ensemble of data with a standard deviation of 0.2%.

In conjunction with the expanding national program of research and development in geothermal energy, White and Williams (27), ERDA (2), it becomes increasingly more important to have access to a variety of thermophysical data concerning brines. In this connection the viscosity plays an important role, because it is needed to calculate pressure losses in wells, ducts, and channels, and, even more importantly, it is essential for the determination of porosities of rocks. Finally, the possibility that Walden's rule, Erdey-Grúz (3), may be applicable, opens the prospect of replacing the measurement of equivalent electric conductance by the, usually simpler, measurement of viscosity.

In order to make a contribution to the solution of this problem, we have organized a program of measurement of the viscosity of a number of brines as a function of pressure, temperature, and composition. The measurements are performed in a slightly modified oscillating-disk viscometer, Kestin and Moszynski (13) and Kestin and Leidenfrost (10), which has been successfully

used in the past for the determination of the viscosity of compressed water, Moszynski (19), as well as superheated steam, Kestin and Wang (16) and Kestin and Richardson (14).

It is natural that a program of measurements on a variety of ionic solutions should start with solutions of sodium chloride. The present paper contains a very preliminary account of our results in the range of temperatures $10\text{--}40^\circ\text{C}$, in the pressure range $0\text{--}30.0$ MPa ($\text{MPa} = 10^6\text{N/m}^2 = 10\text{ bar} = 145.04\text{ psi}$), and over a range of compositions up to about 90% of saturation.

The viscosity of sodium chloride solutions has been measured by a number of investigators, notably by Kaminsky (5), Suryanarayana and Venkatesan (24), Korosi and Fabuss (17), Lengyel (18), Ostroff, Snowden, and Woessner (20), and Werblan, Rotowska, and Minc (26). Data are also listed in the book by Stokes and Mills (23). The measurements available in the literature cover the full range of compositions and a range of temperatures up to 150°C ; they all seem to have been performed by the capillary-flow method, and almost exclusively with the aid of an Ubbelohde-Rankine type of viscometer. However, as far as pressures are concerned, we were unable to locate any measurements at pressures higher than about 1.4 MPa, Potter, Shaw, and Haas (21). The fact that ours seem to be the first measurements which throw light on the effect of pressure on the viscosity of concentrated ionic solutions has provided the motivation for the preparation of this preliminary account. A more detailed report is reserved for a future date.

2. The Method

The data on the viscosity of NaCl–H₂O solutions described in this paper have been obtained in an oscillating-disk instrument.

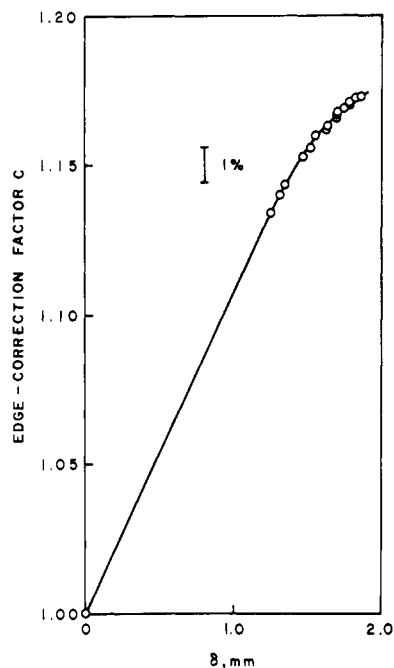


Figure 1. Calibration curve.

The theory of the measurement has been perfected earlier by Kestin and Wang (15); Kestin, Leidenfrost, and Liu (12); and Kestin and Leidenfrost (11). In the present series we have employed the relative method, and based the required calibration on the viscosity of water at 0.1 MPa and over the range of temperatures covered.

The oscillating system has been completely changed, because its characteristics must be matched to the properties of brines which are considerably different from those of a gas or steam. We use a natural period of $T_0 \approx 16$ s, and a stainless steel disk of $R = 33.9725$ mm radius and $d = 3.2131$ mm thickness between two fixed plates of wide spacing $b = 2.9782$ mm. In this manner, no correction for differential thermal expansion is required. The period, T , and the logarithmic decrement, Δ , are measured electronically with the aid of a new device; the details of this new design are described elsewhere (9).

We have used three compositions, namely $c = 2.600$ m, 4.132 m and 5.614 m, where $m = \text{mol/kg of H}_2\text{O}$ (1 mol of NaCl = 58.44 g). In order to keep a running check on the correct operation of the instrument, we determine several points on the calibration curve with reference to water between fillings. These check-points will be characterized in section 4, and it is clear that our procedure is equivalent to periodically remeasuring the viscosity of pure water.

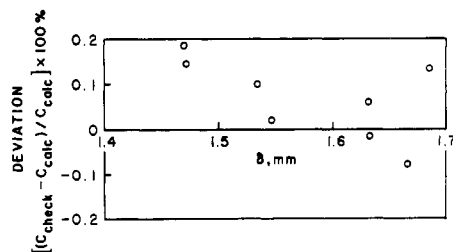


Figure 2. Departure of check-points from eq 5. Standard deviation 0.11%.

3. Calibration

We based the calibration on our own still unpublished, critical correlation of the viscosity of water as a function of temperature at atmospheric pressure. In turn, this is normalized at 20 °C with respect to the value

$$\mu_{20} = 1002 \mu\text{Pa s} \quad (1)$$

(1 Pa s = 1 N s m⁻² = 1 kg s⁻¹ m⁻¹ = 10 P) obtained by Swindells (25) at the National Bureau of Standards. The correlation can be summarized by the equation

$$\log \left(\frac{\mu}{\mu_{20}} \right) = \frac{20 - t}{t + 96} \{ 1.2378 - 1.303 \times 10^{-3}(20 - t) + 3.06 \times 10^{-6}(20 - t)^2 + 2.55 \times 10^{-8}(20 - t)^3 \} \quad (2)$$

valid for -8 °C < t < 150 °C and $P = 0.1$ (or saturation) MPa.

A comparison between this correlation and the recently published international representation of the viscosity of water substance prepared independently by the International Association for the Properties of Steam (4) shows very good agreement. At temperatures $t < 40$ °C the largest discrepancy does not exceed $\pm 0.1\%$. We note, however, for future reference that near the end of the range (150 °C), the difference reaches 0.3%. We believe that eq 2 is more precise, partly because it covers a more restricted range.

Copies of the release may be obtained by writing to Dr. H. J. White, Jr., Secretary IAPS, National Bureau of Standards, Washington, D.C., 20234, or to Professor J. Kestin, President IAPS 1974–1976, Box D, Brown University, Providence, RI 02912.

The calibration curve of the viscometer is shown in Figure 1. The densities required for the calculation of the boundary-layer thickness

$$\delta = (\mu T_0 / 2\pi\rho)^{1/2} \quad (3)$$

Table I. Test of Reproducibility (Distilled Water, Three Fillings)

Temp t , °C	Period T , s	Decrement Δ	Exptl	Viscosity, μ $\mu\text{Pa s}$		Difference %
				Corrected to nominal t		
25.37	17.155 05	0.0508 51	885.0	884.4	+0.02	
25.45	17.155 79	0.0507 99	883.6	884.6	+0.05	
25.27	17.155 55	0.0509 02	886.4	883.8	-0.05	
25.28	17.154 29	0.0509 02	886.5	884.1	-0.01	
25.36	17.159 96	0.0509 34	887.1	886.3	+0.24	
25.51	17.161 54	0.0506 79	880.0	882.2	-0.23	

$d\mu/dt = -20.0 \mu\text{Pa s } ^\circ\text{C}^{-1}$; standard deviation 0.14%. Viscosity at $t = 25.4$ °C from eq 2: 882.1 $\mu\text{Pa s}$; from above table 884.2 $\mu\text{Pa s}$. Difference between measurement and correlation: 0.26%.

Table II. Viscosity of Distilled Water

Pressure, <i>P</i> , MPa	Temp, <i>t</i> , °C	Viscosity, μ , μ Pas		Pressure, <i>P</i> , MPa	Temp, <i>t</i> , °C	Viscosity, μ , μ Pas	
		Exptl	Corrected to t_{nom}			Exptl	Corrected to t_{nom}
a. At 10.6 °C				c. At 22.5 °C			
0.10	10.54	1289.6	1287.2	24.03	22.71	932.6	937.2
3.45	10.52	1285.0	1282.2	27.61	22.76	930.6	936.3
7.06	10.54	1280.4	1278.1	30.99	22.78	929.4	935.7
10.44	10.54	1277.6	1275.2	20.51	22.70	933.8	938.4
13.89	10.55	1272.9	1271.1	13.68	22.70	935.8	940.3
17.58	10.57	1269.2	1268.0	7.00	22.69	937.7	941.8
20.78	10.59	1265.0	1264.5	0.10	22.68	940.2	944.2
24.23	10.60	1262.2	1262.2	0.10	22.77	938.0	944.0
27.82	10.61	1259.6	1260.0	$d\mu/dt = -22.3 \mu\text{Pa s } ^\circ\text{C}^{-1}$			
31.13	10.62	1253.9	1254.6	d. At 30 °C			
13.89	10.61	1273.2	1273.4	0.10	29.94	799.6	798.5
0.10	10.52	1291.3	1288.2	1.72	29.96	799.5	798.8
$d\mu/dt = -36.9 \mu\text{Pa s } ^\circ\text{C}^{-1}$				3.55	30.00	798.0	797.9
b. At 17.3 °C				6.93	30.09	796.3	797.8
0.10	17.19	1077.4	1074.3	10.34	30.06	796.0	796.9
1.76	17.23	1074.3	1072.5	10.82	30.03	795.8	796.4
3.55	17.27	1071.8	1071.0	17.41	30.03	796.6	797.0
7.00	17.33	1068.2	1069.1	20.85	30.11	794.8	796.7
10.44	17.36	1065.0	1066.6	24.16	30.14	794.1	796.5
13.89	17.39	1063.2	1065.7	27.54	30.06	795.5	796.5
17.34	17.42	1059.7	1063.1	31.06	30.05	795.1	796.0
20.51	17.46	1058.2	1062.5	14.04	29.93	798.9	797.7
24.03	17.49	1055.8	1061.1	0.10	29.83	801.5	798.6
27.47	17.52	1054.0	1059.9	$d\mu/dt = -17.0 \mu\text{Pa s } ^\circ\text{C}^{-1}$			
30.99	17.55	1052.3	1059.0	e. At 38.7 °C			
27.27	17.52	1054.4	1060.4	0.10	38.72	669.8	670.0
20.51	17.50	1057.1	1062.4	1.65	38.69	670.5	670.4
13.79	17.46	1062.3	1066.6	3.55	38.63	670.8	669.8
6.93	17.45	1065.2	1069.4	7.00	38.67	670.8	670.4
0.10	17.47	1069.3	1073.8	10.44	38.65	671.5	670.9
$d\mu/dt = -27.3 \mu\text{Pa s } ^\circ\text{C}^{-1}$				13.92	38.66	671.6	671.1
c. At 22.5 °C				17.34	38.65	671.6	671.0
0.10	22.43	945.9	944.3	20.78	38.61	672.2	671.0
1.76	22.45	944.7	943.6	24.23	38.62	673.5	672.4
3.55	22.46	943.4	942.5	27.75	38.62	673.5	672.5
6.93	22.51	942.0	942.2	31.13	38.62	673.4	672.4
10.44	22.55	939.9	941.0	14.10	38.50	672.8	670.3
13.89	22.57	937.7	939.3	0.10	38.41	674.2	670.6
17.35	22.61	937.0	939.4	$d\mu/dt = -12.7 \mu\text{Pa s } ^\circ\text{C}^{-1}$			
20.58	22.64	934.5	937.7				

have been taken from the correlation equation

$$\frac{\rho}{[\text{kg/m}^3]} = \left[\sum_{i=0}^5 a_i t^i \right] / (1 + bt), \quad (4)$$

$(0 < t < 150 \text{ } ^\circ\text{C}, P = 0.1 \text{ MPa})$

with $a_0 = 999.8396$, $a_1 = 18.224\ 944$, $a_2 = -7.922\ 210 \times 10^{-3}$, $a_3 = -55.448\ 46 \times 10^{-6}$, $a_4 = 149.7562 \times 10^{-9}$, $a_5 = -393.2952 \times 10^{-12}$, $b = 18.159\ 725 \times 10^{-3}$, and t in °C, due to Kell (δ). At higher pressures, we use the modification introduced by Kell and Whalley (δ).

The calibration curve can be correlated by the equation

$$C = 1.0000 + 1.04546\delta + 2.16908\delta^2 - 14.9540\delta^3 \quad (5)$$

with a standard deviation of 0.06%. The diagram in Figure 2 shows the deviations of the various check-points discussed in section 2 from this mean curve; it is seen that they do not depart from it by more than $\pm 0.2\%$.

4. Reproducibility

Table I contains data which allow us to assess the reproducibility of our measurements. The table lists the data for six

measurements taken on distilled water at atmospheric pressure at close temperatures but with three different fillings. The measurements have been corrected to a nominal temperature of 25.4 °C by the use of a local correction of $-20.0 \mu\text{Pa s } ^\circ\text{C}^{-1}$ deduced from eq 2. The maximum deviation of the individual results is $\pm 0.2\%$, and the standard deviation is 0.14%. We are thus justified in assuming that our precision over the present temperature range is $\pm 0.2\%$. The action of pressurizing the instrument does not adversely affect the precision of our measurements.

5. Verification and Accuracy

Before engaging in measurements on ionic solutions, we have determined the viscosity of triple-distilled water along five isotherms and over the whole range of pressures. The results are given in Table II and shown graphically in Figure 3; they are compared there with the international correlation. The two sets of data agree remarkably well. On the one hand, this gives us an assurance that our instrument is functioning correctly. On the other hand, since our precision is probably superior to that of the data on which the international formulation has been based,

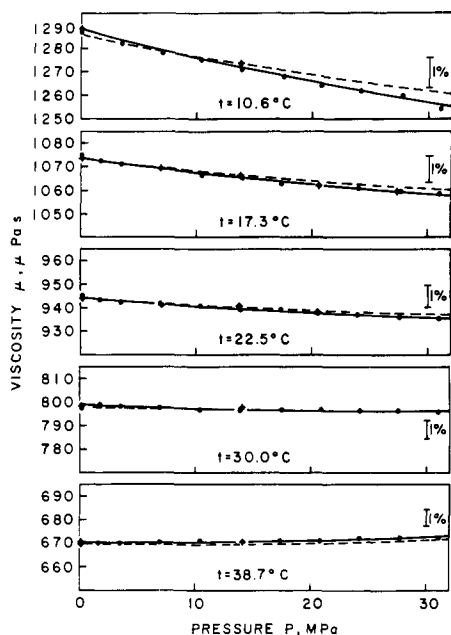


Figure 3. The viscosity of compressed water: —, present measurements; - - -, international correlation by IAPS (4); ●, pressure increasing; ●, pressure decreasing.

we may also interpret this result as reinforcing our confidence in the equation proposed by the International Association for the Properties of Steam. We estimate that our data are accurate to 0.3%.

An examination of Figure 3 confirms the already known fact that the pressure coefficient of the viscosity of water can be both positive and negative, changing sign from negative to positive at about 32.7 °C (see also Figure 7 infra). This compares with a transition temperature of 35 °C obtained earlier by Moszynski (19) in the same instrument but with a different suspension system—a sphere in his case. In his summary article, Kell (7) indicates a transition temperature of 33.5 °C based on data obtained by Bett and Cappi (1).

6. Experimental Results

The results of our measurements are listed in Tables III–V. They are also depicted graphically in Figure 4a–c. The results have been reduced to a convenient nominal temperature by applying a correction based on locally fitting the experimental results to an Arrhenius–Andrade expression of the form

$$\mu = A \exp[B/T] \quad (2a)$$

which is sufficiently accurate for our purpose.

The density of the brines was calculated with the aid of an empirical equation of state worked out by Rowe and Chou (22) and believed to be accurate to within 0.15%. The composition, c , was determined by careful weighing which guaranteed four significant figures, as quoted. To check this accuracy we determined the density of each solution at room temperature with the aid of a pycnometer accurate to 0.01% and compared it with the empirical equation of Rowe and Chou (22). The comparison contained in Table VI establishes confidence in this measurement.

An examination of Figure 4a–c suggests that the tendency to negative pressure coefficients, characteristic of the solvent at the lower temperatures, seems to disappear with increasing molality. At $c = 5.614 \text{ m}$ (about 0.9 of saturation in our temperature range) the pressure coefficient has become nearly constant and almost independent of temperature.

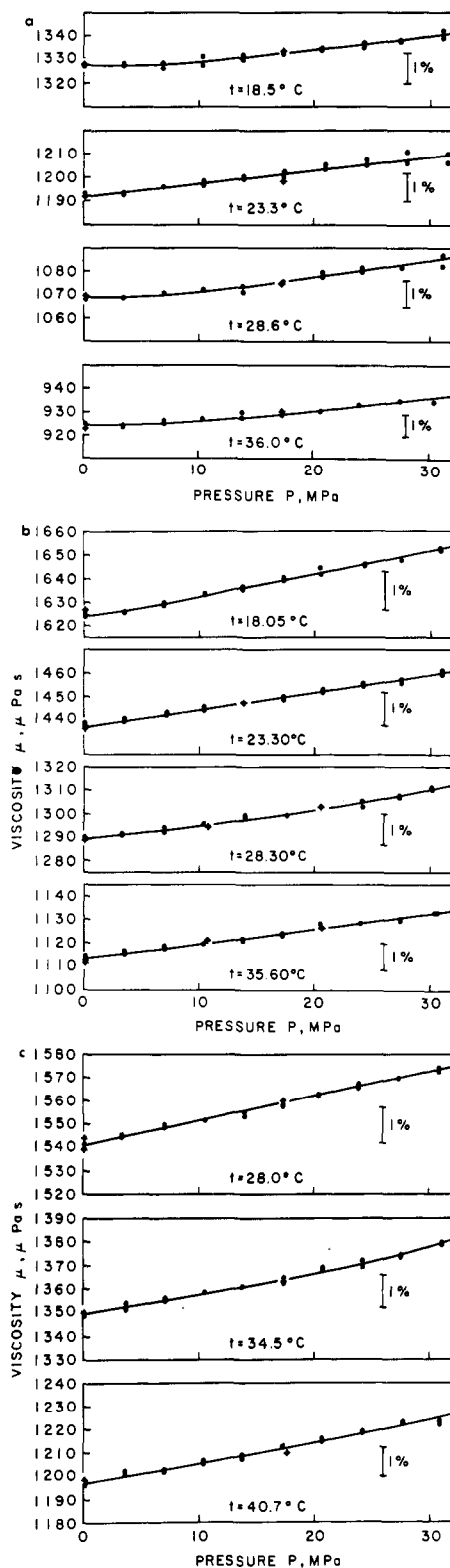


Figure 4 (a). Viscosity of NaCl–H₂O solution 1, $c = 2.600 \text{ m}$. (b) Viscosity of NaCl–H₂O solution 2, $c = 4.132 \text{ m}$. (c) Viscosity of NaCl–H₂O solution 3, $c = 5.614 \text{ m}$.

7. Correlation

In order to make our data quickly available for practical use, we provide a purely empirical correlation which is adequate for interpolation.

First, we notice that the relative viscosity

$$\mu_r(c, t) = \mu(t, c) / \mu(t, 0) \quad (6)$$

Table III. Viscosity of NaCl Solution 1, $c = 2.600\text{ m}$

Pressure, P , MPa	Temp, t , °C	Viscosity, μ , $\mu\text{Pa s}$		Pressure, P , MPa	Temp, t , °C	Viscosity, μ , $\mu\text{Pa s}$	
		Exptl	Corrected to t_{nom}			Exptl	Corrected to t_{nom}
a. At 18.5 °C				c. At 28.5 °C			
0.10	18.73	1320.6	1327.3	0.10	28.53	1068.7	1069.3
0.10	18.72	1320.6	1327.1	0.10	28.52	1067.4	1067.8
3.48	18.69	1322.1	1327.6	3.55	28.55	1067.6	1068.7
3.48	18.67	1323.2	1328.2	3.55	28.55	1067.5	1068.6
6.89	18.56	1327.0	1328.8	6.99	28.57	1069.0	1070.6
6.89	18.56	1324.6	1326.3	6.99	28.57	1068.9	1070.5
10.34	18.55	1326.1	1327.5	10.44	28.60	1070.0	1072.1
10.34	18.54	1330.4	1331.5	10.44	28.59	1070.2	1072.1
13.89	18.53	1331.0	1332.0	13.92	28.62	1068.4	1071.1
13.89	18.53	1329.3	1330.0	13.92	28.61	1071.1	1073.5
17.27	18.52	1331.8	1332.3	17.41	28.65	1072.5	1075.8
17.27	18.50	1332.6	1332.7	17.41	28.64	1072.6	1075.6
20.65	18.54	1332.3	1333.4	20.78	28.67	1076.0	1079.8
20.65	18.51	1334.3	1334.5	20.78	28.66	1074.2	1077.8
24.23	18.52	1336.6	1337.3	24.16	28.68	1077.6	1081.6
24.23	18.51	1334.8	1335.1	24.16	28.66	1076.1	1079.7
27.40	18.46	1339.8	1338.7	27.61	28.69	1077.7	1081.8
27.40	18.46	1338.7	1337.4	27.61	28.67	1078.4	1082.1
30.99	18.49	1342.5	1342.3	31.06	28.70	1082.8	1087.2
30.99	18.48	1339.8	1339.2	31.06	28.67	1078.5	1082.3
17.34	18.26	1341.1	1333.9	17.27	28.54	1074.3	1075.1
0.10	18.14	1338.4	1327.9	0.10	28.44	1071.5	1069.6
$d\mu/dt \approx -29.4\ \mu\text{Pa s } ^\circ\text{C}^{-1}$				$d\mu/dt \approx -22.2\ \mu\text{Pa s } ^\circ\text{C}^{-1}$			
b. At 23.3 °C				d. At 36.0 °C			
0.10	23.36	1190.7	1192.2	0.10	35.89	926.2	924.2
0.10	23.35	1191.5	1192.7	0.10	35.96	926.0	925.3
3.48	23.34	1191.4	1192.4	3.53	36.06	922.5	923.5
3.48	23.31	1193.3	1193.6	3.53	36.08	922.8	924.2
6.93	23.32	1195.4	1195.9	7.04	36.16	923.8	926.7
6.93	23.30	1195.8	1195.9	7.04	36.19	921.7	925.2
10.37	23.31	1198.5	1198.6	10.44	36.24	922.8	927.0
10.37	23.28	1196.9	1196.4	10.44	36.25	922.6	927.1
13.96	23.30	1200.6	1200.7	13.92	36.27	925.3	930.1
13.96	23.27	1200.2	1199.5	13.92	36.26	923.0	927.7
17.41	23.29	1202.6	1202.4	17.41	36.30	923.4	928.9
17.41	23.27	1201.8	1201.1	17.41	36.30	924.7	930.1
20.99	23.29	1206.1	1205.8	20.65	36.32	924.7	930.5
20.99	23.26	1204.4	1203.4	20.65	36.32	925.0	930.8
24.58	23.28	1208.4	1207.8	23.97	36.35	926.9	933.2
24.58	23.24	1206.6	1205.2	23.97	36.35	927.1	933.3
28.02	23.28	1211.6	1211.0	27.47	36.38	928.4	935.2
28.02	23.24	1207.5	1206.3	27.47	36.36	928.6	935.1
31.47	23.26	1211.5	1210.4	30.51	36.35	928.4	934.7
31.47	23.24	1207.7	1206.1	30.51	36.36	928.1	934.5
17.34	23.00	1208.9	1198.6	17.41	36.24	925.6	930.0
0.10	22.80	1204.7	1192.0	0.10	36.11	921.1	923.0
$d\mu/dt \approx -25.5\ \mu\text{Pa s } ^\circ\text{C}^{-1}$				$d\mu/dt \approx -18.0\ \mu\text{Pa s } ^\circ\text{C}^{-1}$			

i.e., the ratio of the viscosity of the solution, $\mu(t,c)$, extrapolated to $P = 0$, to that of distilled water, $\mu(t,0)$, at the same temperature and pressure is only a weak function of temperature in our limited temperature range; this is clearly illustrated in Figure 5. We represent these data by the equation

$$\mu_r(c,t) = \phi(c) + \psi(c)t \quad (7)$$

and propose to put

$$\phi(c) = 1.0000 + 0.00800c + 0.03886c^2 - 0.00242c^3 \quad (8)$$

and

$$\psi(c) = (0.0014/^\circ\text{C})ce^{-0.096c^2} \quad (9)$$

The diagram of Figure 6 represents the deviation plot of all our low-pressure data from eq 7; it shows that the interpolating equation represents the data with a standard deviation of 0.14% and a maximum deviation of 0.31%, that is with practically the same uncertainty as is claimed for the measurement itself.

It is not convenient to represent the effect of pressure by employing the theoretically preferable variable-density, owing to the very low compressibility of our liquids. Thus, the effect of pressure has been represented by a factor

$$1 + a(c,t)P \quad (10)$$

where a is plotted in Figure 7; it can be correlated by the equation

$$a(c,t) = \alpha'[(1 - e^{-c/2}) + (\alpha + \beta t + \gamma t^2)e^{-c/2}] \quad (11)$$

Table IV. Viscosity of NaCl Solution 2, $c = 4.132 \text{ m}$

Pressure, P_i MPa	Temp, t_i °C	Viscosity, μ , $\mu\text{Pa s}$		Pressure, P_i MPa	Temp, t_i °C	Viscosity, μ , $\mu\text{Pa s}$	
		Exptl	Corrected to t_{nom}			Exptl	Corrected to t_{nom}
a. At 18.05 °C				c. At 28.3 °C			
0.10	17.99	1631.9	1629.7	0.10	27.97	1298.1	1289.1
0.10	18.01	1628.5	1627.1	0.10	27.94	1299.4	1289.5
0.10	18.00	1628.3	1626.5	3.36	27.93	1300.6	1290.5
3.45	18.06	1625.4	1625.8	3.36	27.93	1301.0	1291.0
3.45	18.05	1625.7	1625.7	6.98	27.99	1300.8	1292.3
6.93	18.07	1628.8	1629.5	6.98	28.00	1302.1	1293.9
6.93	18.07	1628.4	1629.1	10.40	28.05	1302.3	1295.3
10.42	18.08	1632.5	1633.6	10.40	28.05	1302.0	1295.2
10.42	18.08	1632.0	1633.1	13.96	28.12	1303.3	1298.4
13.79	18.06	1634.5	1634.9	13.96	28.14	1302.4	1298.0
13.79	18.06	1636.8	1637.2	17.57	28.45	1295.5	1299.4
17.28	18.06	1638.9	1639.3	17.57	28.48	1294.3	1299.1
17.28	18.06	1640.2	1640.6	20.51	28.53	1297.1	1303.3
20.50	18.05	1641.7	1641.7	20.51	28.54	1293.5	1299.9
20.50	18.05	1644.4	1644.4	24.09	28.59	1294.8	1302.8
24.30	18.06	1646.0	1646.4	24.09	28.68	1295.2	1305.4
24.30	18.05	1645.7	1645.7	27.30	28.75	1294.6	1307.0
27.47	18.06	1648.0	1648.4	27.30	28.77	1294.6	1307.5
27.47	18.06	1648.1	1648.5	30.71	28.82	1296.9	1311.1
30.78	18.06	1651.6	1652.0	30.71	28.83	1295.6	1310.1
30.78	18.06	1652.9	1653.3	20.54	28.77	1290.6	1302.9
0.10	18.01	1625.3	1623.9	10.72	28.75	1282.1	1294.3
0.10	18.02	1626.2	1625.1	0.10	28.72	1276.9	1288.5
$d\mu/dt \approx -36.0 \mu\text{Pa s } ^\circ\text{C}^{-1}$				$d\mu/dt \approx -27.3 \mu\text{Pa s } ^\circ\text{C}^{-1}$			
b. At 23.3 °C				d. At 35.6 °C			
0.10	23.02	1447.5	1438.6	0.10	35.51	1115.0	1113.0
0.10	23.09	1444.1	1437.5	0.10	35.51	1115.6	1113.7
0.10	23.11	1443.9	1437.9	0.10	35.53	1115.6	1114.0
3.54	23.18	1443.7	1439.8	0.10	35.56	1112.6	1111.6
3.54	23.20	1444.0	1440.1	0.10	35.58	1112.7	1112.3
7.09	23.26	1443.6	1442.2	3.47	35.56	1115.8	1114.9
7.09	23.27	1443.6	1442.7	3.47	35.56	1117.7	1116.8
10.41	23.35	1444.2	1445.7	7.00	35.60	1117.8	1117.7
10.41	23.35	1442.3	1443.9	7.00	35.60	1117.1	1117.0
13.89	23.41	1442.7	1446.3	7.00	35.60	1118.3	1118.2
13.89	23.42	1443.8	1447.7	10.38	35.62	1119.2	1119.7
17.34	23.48	1443.3	1448.9	10.38	35.63	1118.9	1119.5
17.34	23.48	1444.2	1449.8	13.83	35.65	1120.5	1121.7
20.71	23.53	1445.1	1452.4	13.83	35.64	1120.1	1121.1
20.71	23.53	1445.5	1452.7	17.23	35.66	1122.7	1124.0
24.23	23.59	1446.5	1455.6	17.23	35.66	1122.5	1123.9
24.23	23.59	1445.9	1455.0	20.50	35.69	1126.0	1128.0
27.54	23.63	1447.1	1457.3	20.50	35.69	1126.1	1128.1
27.54	23.63	1445.6	1455.9	24.03	35.69	1126.4	1128.4
30.98	23.68	1447.8	1459.6	24.03	35.68	1126.6	1130.1
30.98	23.67	1449.5	1461.1	27.42	35.70	1128.0	1130.1
13.88	23.59	1438.5	1447.5	27.42	35.70	1127.5	1129.7
0.10	23.62	1426.6	1436.7	30.65	35.70	1131.1	1133.3
$d\mu/dt \approx -31.4 \mu\text{Pa s } ^\circ\text{C}^{-1}$				$d\mu/dt \approx -22.3 \mu\text{Pa s } ^\circ\text{C}^{-1}$			

with

$$\left. \begin{aligned} \alpha' &= 0.00075 (\text{MPa})^{-1} \\ \alpha &= -1.791, \quad \beta = 0.07733 (\text{°C})^{-1} \\ \gamma &= -0.6827 \times 10^{-3} (\text{°C})^{-2} \end{aligned} \right\} \quad (12)$$

The totality of our results is thus given by the interpolation formula

$$\mu(t, c, P) = (1002 \mu\text{Pa s}) 10^E (\phi + \psi t) (1 + aP) \quad (13)$$

where

$$E = \frac{20 - t}{t + 96} \{ 1.2378 - 1.303 \times 10^{-3} (20 - t) + 3.06 \times 10^{-6} (20 - t)^2 + 2.55 \times 10^{-8} (20 - t)^3 \} \quad (t \text{ in } ^\circ\text{C}) \quad (14)$$

and ϕ , ψ , and a are given by eq 8, 9, and 11, respectively.

The deviation plot of Figure 8 convinces us that eq 13 represents our data with a standard deviation of 0.18% or a maximum deviation of 0.51%. The preceding empirical correlation

Table V. Viscosity of NaCl Solution 3, $c = 5.614\text{ m}$

Pressure, P , MPa	Temp, t , °C	Viscosity, μ , $\mu\text{Pa s}$	
		Exptl	Corrected to t_{nom}
a. At 28 °C			
0.10	27.87	1545.2	1541.0
0.10	27.86	1546.2	1541.5
3.34	27.86	1549.2	1544.7
3.34	27.86	1549.7	1545.2
7.03	27.91	1551.3	1548.5
7.03	27.92	1552.0	1549.5
10.49	28.05	1549.7	1551.2
10.49	28.07	1549.7	1551.9
13.96	28.13	1548.9	1553.1
13.96	28.15	1550.4	1555.2
17.34	28.21	1552.1	1558.8
17.34	28.22	1551.4	1558.3
20.44	28.26	1553.5	1562.1
20.44	28.27	1554.0	1562.5
23.85	28.31	1557.5	1567.5
23.85	28.30	1555.6	1565.2
27.34	28.34	1559.0	1569.9
27.34	28.33	1559.2	1569.8
30.85	28.36	1561.2	1572.7
30.85	28.35	1563.2	1574.4
17.34	28.16	1555.1	1560.3
0.10	28.56	1521.0	1539.0
$d\mu/dt \approx -32.2\ \mu\text{Pa s } ^\circ\text{C}^{-1}$			
b. At 34.5 °C			
0.10	34.47	1350.4	1349.6
0.10	34.46	1349.6	1348.6
3.65	34.43	1356.0	1354.3
3.65	34.42	1353.2	1350.9
7.07	34.45	1356.4	1355.0
7.07	34.45	1357.5	1356.0
10.51	34.48	1359.1	1358.5
10.51	34.46	1359.5	1358.3
13.82	34.45	1362.2	1360.9
13.82	34.45	1362.4	1361.0
17.41	34.48	1365.4	1364.9
17.41	34.47	1363.4	1362.6
20.72	34.50	1367.8	1367.9
20.72	34.49	1369.5	1369.2
24.16	34.51	1368.9	1369.2
24.16	34.50	1371.9	1371.9
27.54	34.51	1373.2	1373.3
27.54	34.50	1374.2	1374.2
31.13	34.53	1379.0	1379.7
31.13	34.51	1378.4	1378.8
17.34	34.38	1366.5	1363.3
0.10	34.15	1359.1	1349.7
$d\mu/dt \approx -27.0\ \mu\text{Pa s } ^\circ\text{C}^{-1}$			
c. At 40.7 °C			
0.10	40.71	1196.0	1196.1
0.10	40.68	1198.6	1198.2
3.63	40.71	1202.0	1202.4
3.63	40.70	1200.4	1200.4
7.01	40.71	1201.5	1201.7
7.01	40.71	1202.9	1203.1
10.40	40.68	1207.0	1206.5
10.40	40.69	1205.4	1205.1
13.82	40.71	1206.9	1207.0
13.82	40.70	1209.2	1209.1
17.34	40.73	1211.4	1212.1
17.34	40.72	1212.5	1212.9
20.65	40.72	1215.2	1215.6
20.65	40.71	1214.0	1214.1
24.16	40.70	1218.1	1218.2
24.16	40.70	1219.3	1219.2
27.68	40.71	1223.0	1223.1
27.68	40.71	1222.2	1222.4
30.85	40.68	1222.1	1221.6
30.85	40.68	1224.0	1223.4
17.68	40.57	1214.9	1209.7
0.10	40.47	1203.6	1198.3
$d\mu/dt \approx -22.9\ \mu\text{Pa s } ^\circ\text{C}^{-1}$			

Table VI. Measured and Calculated Density at Room Temperature

Temp t , °C	Density ρ , kg m^{-3}			
	Measd	From Rowe and Chou (22)	Difference, %	
Solution 1	26.0	1.092 78	1.092 79	0.00
Solution 2	23.8	1.142 55	1.142 43	0.01
Solution 3	20.0	1.187 16	1.186 65	0.04

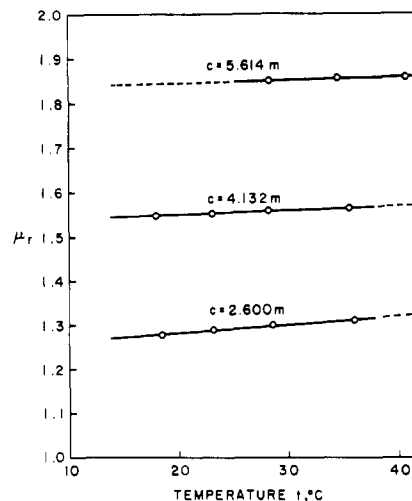


Figure 5. The low-pressure data as a function of temperature with composition as a parameter.

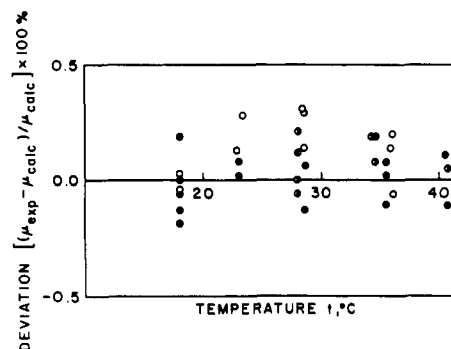


Figure 6. Deviation plot for eq 7 with eq 8 and 9 (standard deviation 0.14%; maximum deviation 0.31%); ○, solution 1, $c = 2.600\text{ m}$; ◐, solution 2, $c = 4.132\text{ m}$; ●, solution 3, $c = 5.614\text{ m}$.

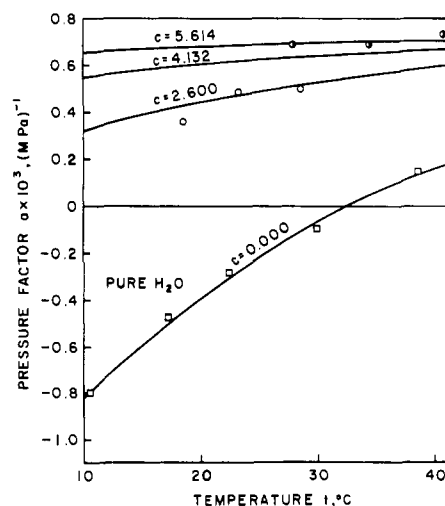


Figure 7. The pressure factor a as a function of temperature, t , and molality, c .

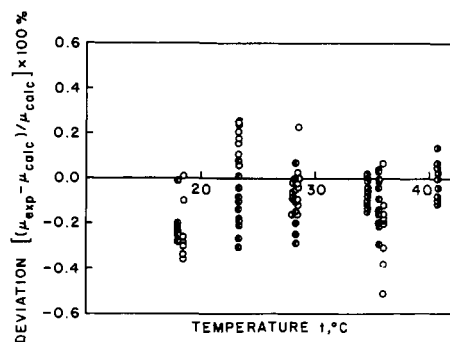


Figure 8. Deviation plot for the ensemble of our data referred to eq 11 (standard deviation 0.18%; maximum deviation of 0.51%): solution 1, $c = 2.600$ m; solution 2, $c = 4.132$ m; solution 3, $c = 5.614$ m.

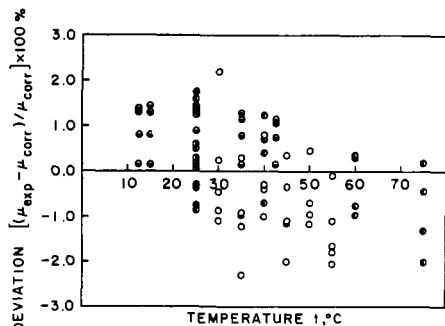


Figure 9. Results from the literature (standard deviation 1.1%; maximum deviation 2.25%): O, Suryanarayana et al. (24); ●, Korosi and Fabuss (17); ●, Ostroff et al. (20); ●, Kaminsky (5); ●, Werblan et al. (26); baseline represents eq 13.

has not lost much of the accuracy of our measurements, and is certainly adequate for preliminary geothermal applications.

8. Other Results

The deviation plot of Figure 9 shows that the literature data on the viscosity of NaCl-H₂O solutions at low pressures agree well with our new measurements, even though their combined standard deviation from our correlation is 1.1%, the maximum deviation reaching 2.25%.

Of particular interest are the data and the correlation due to Korosi and Fabuss (17). The maximum deviation of these data from our correlation does not exceed 1.5%. Without going into detailed comparisons, it may be useful to restate here that their low-pressure correlation has the form

$$\log \mu_r(c,t) = A_0 + B_0 \log \mu(0,t), \text{ with } \mu \text{ in cP} \quad (15)$$

$$A_0 = A_1c + A_2c^2 + A_3c^3$$

$$B_0 = B_1c + B_2c^2 + B_3c^3 \quad (16)$$

with $A_1 = 0.0355$, $A_2 = 0.00231$, $A_3 = -0.00003$, $B_1 = -0.04753$, $B_2 = 0.01598$, and $B_3 = -0.00194$.

A direct comparison with our eq 11 would show a maximum discrepancy of 2% at $c \approx 1$ m, quite good agreement at $c \approx 3$ m, but a diverging tendency for $c > 4$ m. For example, at $t = 40$ °C and $c \approx 6$ m, eq 15 gives a value which is higher by 6% than eq 11 and by 6–7% compared with the direct measurement.

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